05th Aug 2020

B.Sc. Part II 4th Paper Diff. Equations

LDECC (contd.)

Q. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$.

Find the value of y when $x = \frac{\pi}{2}$ if it is given that $y = 3$ and $\frac{dy}{dx} = 0$ when $x = 0$.

Soln. The given equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = -37\sin 3x$$

$$\Rightarrow (D^2 + 2D + 10)y = -37\sin 3x.$$

For CF.

$$D^2 + 2D + 10 = 0$$

$$\Rightarrow (D+1)^2 = 9i^2 \Rightarrow D+1 = \pm 3i$$

$$\Rightarrow D = -1 \pm 3i$$

$$\therefore CF = c_1 e^{(-1+3i)x} + c_2 e^{(-1-3i)x}$$

$$\Rightarrow CF = c_1 e^{-x} \cdot e^{3ix} + c_2 e^{-x} \cdot e^{-3ix}$$

$$\Rightarrow CF = c_1 e^{-x}(\cos 3x + i\sin 3x) + c_2 e^{-x}(\cos 3x - i\sin 3x)$$

$$= e^{-x}[(c_1 + c_2)\cos 3x + (c_1 - c_2)i\sin 3x]$$

$$\Rightarrow CF = e^{-x}(A\cos 3x + B\sin 3x)$$

For PI, $y = \frac{1}{D^2 + 2D + 10} \cdot (-37\sin 3x)$

$$\Rightarrow y = \frac{-37}{D^2+2D+10} \sin 3x$$

$$\Rightarrow y = \frac{-37}{-3^2+2D+10} \sin 3x = \frac{-37}{2D+1} \sin 3x$$

$$\Rightarrow y = \frac{-37}{2D+1} \times \frac{2D-1}{2D-1} \sin 3x$$

$$= \frac{-37(2D-1)}{4D^2-1} \sin 3x$$

$$\Rightarrow y = \frac{-37(2D-1)}{4\times(-3^2)-1} \sin 3x$$

$$\Rightarrow y = \frac{-37(2D-1)}{-37} \sin 3x$$

$$\Rightarrow y = (2D-1)\sin 3x$$

$$\Rightarrow y = 2D(\sin 3x) - \sin 3x = 6\cos 3x - \sin 3x$$

So, the complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = e^{x}(A\cos 3x + B\sin 3x) + 6\cos 3x - \sin 3x \quad \text{---(1)}$$

Differentiating (1) w.r.t x, we get-

$$\frac{dy}{dx} = -e^{-x}(A\cos 3x + B\sin 3x) + e^{-x}(-3A\sin 3x + 3B\cos 3x) - 18\sin 3x - 3\cos 3x \quad \text{---(2)}$$

Given that $y = 3, \frac{dy}{dx} = 0$ when $x = 0$

So, (2) $\Rightarrow 0 = -1(A+0) + 1(0+3B) - 0 - 3$

$\Rightarrow 0 = -A + 3B - 3$

$\Rightarrow A - 3B = -3 \quad \text{---(3)}$

and (1) $\Rightarrow 3 = 1(A+0) + 6 - 0$

$\Rightarrow 3 = A + 6 \Rightarrow A = -3 \quad \text{---(4)}$

using (4) in (3) $\Rightarrow -3 - 3B = -3$

$\Rightarrow B = 0 \quad \text{---(5)}$

Putting the value of A and B in (1), we get-

$y = e^{-x}(-3\cos 3x + 0) + 6\cos 3x - \sin 3x$

$\Rightarrow y = 6\cos 3x - \sin 3x - 3e^{-x}\cos 3x$

Put $x = \frac{\pi}{2}$

$\Rightarrow y = 6\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2} - 3e^{-\frac{\pi}{2}}\cdot\cos\frac{3\pi}{2}$

$= 0 + 1 - 0 = 1$

Hence $y = 1$